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ON THE PROBLEM OF INTEGRALS OF RELATIVE MOTION OF A SYSTEM OF PARTICLES POSSESSING IDENTICAL MASS

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It is stated in the paper [1] by Machil'skii that the three- or many-body problem of the objects of identical masses can be solved in a finite form by separating the equations of motion in terms of relative coordinates. The work contains an obvious error which was pointed out to the author during discussions. Nevertheless the paper was published with an ackowledgement of my "constructive criticism". Moreover, the review journal "Mekhanika" [Mechanics] inserted in its №2, 1973, the author's abstract in which he states that the many-body problem of objects of identical masses is not embraced by the Bruns theorem. This made it necessary to publish the present paper with the analysis of erroneous assumptions contained in [1].

Using the formalized notation the author of [1] arrives at the following assertion. For three vectors η_1, η_2 and η_3 forming a closed vector triangle

$$\eta_1 + \eta_2 + \eta_3 \equiv 0 \tag{1}$$

the identity $\nabla_{\eta_1} + \nabla_{\eta_2} + \nabla_{\eta_3} \equiv 0$ is satisfied. If the projections of the vector η_i on the Cartesian axes are denoted by η_{ix}, η_{iy} and η_{iz}, then the suthor's assertion becomes equivalent to the following statement: for any function $\varphi(\eta_{ix}, \eta_{iy}, \eta_{iz})$ we have, provided that $\eta_{1x} + \eta_{2x} + \eta_{3x} = 0$,

$$\frac{\partial\varphi}{\partial\eta_{1x}} + \frac{\partial}{\partial\eta_{2x}} \quad \frac{\partial\varphi}{\partial\eta_{3x}} = 0$$

and this is not true when e.g. $\varphi = \eta_{1x}$.

The separation of the equations and, consequently, the positive solution of the problem in question, is reduced by the author to making the right-hand side of (1.12) in [1] vanish, i.e. to

$$\nabla_{\eta_1} V_1 + \nabla_{\eta_2} V_2 + \nabla_{\eta_3} V_3 \equiv 0 \tag{2}$$

Here $V_i = -\eta_i^{-1}$, therefore $\Delta_{\eta_i} V_i = \eta_i \eta_i^{-3}$ is the Newtonian force of attraction between two neighboring bodies of equal mass. The relation (2) now becomes

$$\eta_1\eta_1^{-3} + \eta_2\eta_2^{-3} + \eta_3\eta_3^{-3} \equiv 0 \tag{3}$$

The paper in question asserts that (3) follows from (1). This is however a complementary condition. Indeed, considering (1) and (3) together we obtain

$$\eta_1(\eta_1^{-3} - \eta_3^{-3}) + \eta_2(\eta_2^{-3} - \eta_3^{-3}) \equiv 0 \tag{4}$$

Two cases are possible: (a) η_1 and η_2 are collinear, hence η_3 and η_2 are collinear; (b) expressions within the parentheses in (4) vanish identically, i.e. $\eta_1 = \eta_2 = \eta_3$. In the case (a) the three bodies are distributed along a straight line. Let us suppose for definiteness that the body 3 lies between the bodies 1 and 2, i.e. $\eta_1 = \eta_2 + \eta_3$. Assume

$\eta_2 = z\eta_3$, then $\eta_1 = (1 + z)\,\eta_3$. For the collinear distribution of bodies under consideration the formula (3) becomes, in the notation adopted,

$$z^4 + 2z^3 + z^2 + 2z + 1 = 0 \tag{5}$$

In the well-known Euler case of separation of equations we have, for the collinear distribution and equal masses, $z = 1$. Since Eq. (5) has no positive roots and $z = 1$ in particular is not a root of this equation, the condition (2) fails even in the case of an Eulerian motion of bodies with equal masses. In the case (b) the bodies must be distributed at the vertices of an equilateral triangle. This represents the Lagrange solution of the three-body problem.

Note. The well-known K. Stumpff's proof [2] that the Euler and Lagrange cases are the only cases in which the relative motions follow the Keplerian orbits is also carried out using the relative coordinates and has no singularities in the case of bodies of equal mass.

REFERENCES

1. Machil'skii, A. P., Integrals of relative motion of a system of particles of identicall mass. PMM Vol. 36, №5, 1972.
2. Subbotin, M. F., Introduction to Theoretical Astronomy. M., "Nauka", 1968.

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